Unit Title	X Marks the Spot!	**Length of Unit**	5 weeks
Focusing Lens(es)	Equivalence Comparison	**Standards and Grade Level Expectations Addressed in this Unit**	MA10-GR.7-S.2-GLE.1 MA10-GR.7-S.2-GLE.2
Inquiry Questions (Engaging-Debatable):	• Why are there different ways to solve equations? (MA10-GR.7-S.2-GLE.2-IQ.2)		
Unit Strands	Expressions and Equations		
Concepts	Equivalent, expressions, inequalities, properties of operations, addition, subtraction, multiplication, division, factoring, expansion, arithmetic solution strategy, algebraic solution strategy, arithmetic operations, algebraic equations, correctness, algebraic manipulations, operation, both sides, negative number, reverse		

Generalizations **My students will Understand that…**	**Guiding Questions** **Factual**	**Conceptual**
Mathematicians generate equivalent expressions by applying properties of operations to shed light on a problem context and the relationships between quantities. (MA10-GR.7-S.2-GLE.1-EO.a.i, a.ii)	What is an expression? How is it determined that two algebraic expressions are equivalent? (MA10-GR.7-S.2-GLE.1-IQ.2)	How do symbolic transformations affect an expression?(MA10-GR.7-S.2-GLE.1-IQ.1) How does rewriting the expression shed light on how the quantities in the problem are related?
Comparing arithmetic and algebraic solution strategies provides a basis for checking the correctness of algebraic manipulations. (MA10-GR.7-S.2-GLE.2-EO.c.i, c.ii)	What does it mean to solve an equation arithmetically?	How can substituting a value for x to allow for arithmetic operations assist in the solving of algebraic equations? Why do properties of operations work with numbers and variables? (MA10-GR.7-S.2-GLE.2-IQ.1)
Generating equivalent inequalities requires applying the same operations in the same order to both sides of an inequality. (MA10-GR.7-S.2-GLE.2-EO.c.iii, c.iv)	What is an inequality? How is it determined if inequalities written differently are equivalent? How can you graph the solution set of an inequality? What are examples of context that produce inequalities with infinitely or finitely many solutions?	Why is it necessary to carry out operations in the same order to both sides of inequality to generate an equivalent inequality? Why does multiplying or dividing by a negative number reverse the inequality sign? Why is it important to check a value from the solution set of an inequality in the original inequality?

Key Knowledge and Skills: My students will…	*What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.*

- **Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. (MA10-GR.7-S.2-GLE.1-EO.a.i)**
  - Tasks may involve issues of strategy, e.g., by providing a factored expression such as $y(3+x+k)$ and a fully expanded expression $3y+xy+ky$ and requiring students to produce or identify a new expression equivalent to both (such as $y(3+x)+yk$). PARCC
  - Tasks are not limited to integer coefficients. PARCC
  - Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related PARCC
    - Students synthesize their knowledge of operating with rational numbers and operating with expressions to work on the following kinds of problems:
      - **Problem 1:**

$$\frac{1}{3}x+\frac{2}{5}x=\frac{5}{15}x+\frac{6}{15}x$$

$$=x\left(\frac{11}{15}\right)$$

$$=\frac{11}{15}x$$ **(CC.7.EE.1)**

- **Problem 2:**

$$0.2\,(6-x)=\ 1.2-0.2x$$

- Students beginning to work with rational expressions make several common mistaken assertions. Using the expression $\frac{x+5}{5}$, for example, students commonly and mistakenly assert that $\frac{x+5}{5}=x$ **(CC.7.EE.1)**
  - They claim that the 5's in the numerator and the denominator can be "canceled." This approach may be the result of overgeneralizing from correctly equating expressions such as $\frac{3x}{3}=x$, in which the equivalence of $^3/_3$ = 1 is correct because the underlying structure is of the product $^3/_3$ • *x*. **(CC.7.EE.1)**
- Another common mistake is to write $\frac{x+5}{5}=x+1$, making the assumption that the term $^5/_5$ (= 1) is isolated from *x*. **(CC.7.EE.1)**.
  - Instead, students should use the distributive property to write this expression as

$$\frac{x+5}{5}=\frac{1}{5}(x+5)=\frac{1}{5}x+1$$

- Students simplify expressions by expanding multiplication and combining like terms. They apply properties of operations and their experience with rational numbers to write equivalent expressions. **(CC.7.EE.1)**

- For example, students may be asked to simplify the expression **(CC.7.EE.1)**

$$\frac{3}{4}x + \frac{1}{2}(x + 13) - \frac{1}{2}$$

- They use properties such as the distributive property to remove the parentheses: **(CC.7.EE.1)**

$$\frac{3}{4}x + \frac{1}{2}x + \frac{13}{2} - \frac{1}{2}$$

- They will then combine like terms: **(CC.7.EE.1)**

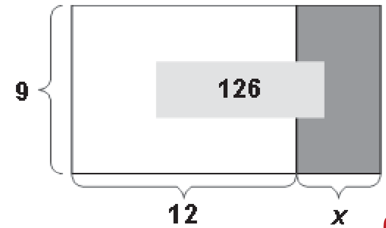
$$\frac{3}{4}x + \frac{1}{2}x + \frac{12}{2}$$

$$\frac{3}{4}x + \frac{2}{4}x + 6$$

$$\frac{5}{4}x + 6$$

(CC.7.EE.1)

- **Solve word problems leading to equations of the form *px + q = r* and *p(x + q) = r*, where *p, q*, and *r* are specific rational numbers. (MA10-GR.7-S.2-GLE.2-EO.c.i)**
  - In this standard, students interpret, write and solve equations that combine multiplication (division) and addition (subtraction). These problems take on the general forms of *px* + *q* = *r* and *p*(*x* + *q*) = *r*. **(CC.7.EE.4a)**
    - For example:
      1. *px* + *q* = *r*: "At the end of each month, the utility company gives every household a rebate of $5 for subscribing to their services. If the company charges $0.20 for every kilowatt-hour of electricity used by the household, how much electricity is used by a family receiving a bill for the month of $68? " Identifying $0.20 per kWh as the unit rate associating the amount of charges per kWh of electricity (see Standard 6.RP.2 and 6.RP.3.b in the Ratio and Proportion, and Percents LT), students express the equation as 0.2*x* − 5 = 68, where *x* represents kWh, and solve it. **(CC.7.EE.4a)**
      2. *p*(*x* + *q*) = *r*: "Patrick has a sunroom of dimensions 12 feet by 9 feet. If he plans to enlarge the room to an area of 126 square feet, how much longer should he extend the length of the sunroom?" Students draw a sketch of Patrick's floor plan and use *x* to represent the length of the extension. **(CC.7.EE.4a)**



(CC.7.EE.4a)

3. Using the area formula, students write the equation associating area with the dimensions of the room, $9(12 + x) = 126$, and then solve for x. **(CC.7.EE.4a)**

- Students solve these types of equations systematically :

Types of Equations	Operations on one or both sides of the equation
1) $0.2x - 5 = 68$	Addition: $(0.2x - 5) + 5 = 68 + 5$ Division: $^{(.2x)}/_{.2} = {}^{(73)}/_{.2}$
2) $9(12 + x) = 126$	Expand: $108 + 9x = 126$ Followed by steps similar to 1) above Or Division: $^{[9(12 + x)]}/_{9} = {}^{126}/_{9}$ Subtraction: $(12 + x) - 12 = (14) - 12$

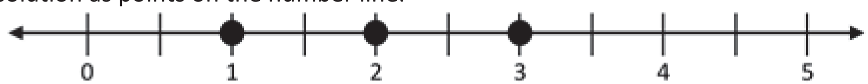
(CC.7.EE.4a)

- While solving these equations, beginning students tend to use strategies that are straight-forward and present fewer opportunities for error. For example, in solving the equation $9(12 + x) = 126$, students tend to expand the expression $9(12 + x)$ rather than dividing both sides of the equation by 9. They may also re-write the equation in another equivalent form that has only whole numbers, for example, $5 + .08x = 0.2$ becomes $500 + 8x = 20$ by multiplying both sides of the equation by 100. **(CC.7.EE.4a)**
- Students are encouraged to explore other methods of finding solutions to an equation using a different first step and compare these solutions to develop adaptive strategies. **(CC.7.EE.4a)**

- **Fluently solve equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. (MA10-GR.7-S.2-GLE.2-EO.c.i)**
  - Students also recognize that for problems leading to equations of both forms $px + q = r$ and $p(x + q) = r$, the unknown may be solved through a series of arithmetic operations on r, without writing and solving an equation in x. They tend to use such strategies of "undoing" wherever applicable. For example, in solving $0.2x - 5 = 68$, students use the sequence of arithmetic calculations, $(68 + 5) \div 0.2$ to solve for x. **(CC.7.EE.4a)**
  - Comparing the arithmetic and algebraic solutions of the same problem step-by-step helps student interpret the action of arithmetic operations in terms of the operations on equivalent equations. When unsure of the correctness of an algebraic manipulation, students use arithmetic problems to check their understanding. For example, when asked to add $^{3}/_{x} + {}^{3}/_{4}$, a student considers if it is equivalent to $^{3}/_{x} + 4$. The student substitutes for x with a value (x = 6) to see if
  - $^{3}/_{6} + {}^{3}/_{4} = {}^{3}/_{10}$. The student thereby sees that the proposed equivalence is incorrect. **(CC.7.EE.4a)**
  - Students interpret real-world problems in which negative rational numbers are involved. For instance, students are asked, "A company reports a monthly average loss of -$300,000. Given that the company still has a balance of $450, 000 in its account, how many months would the company take to accumulate a debt of $1,050,000?" **(CC.7.EE.4a)**

- When solving these equations, students apply their knowledge of negative number operations (see Standard 7.EE.3 in the Rational and Irrational Numbers LT). They also represent the solution for the unknown *x* on the number line. Students develop competencies in solving equations involving *p*, *q* and *r* as rational numbers. They apply rational number operations in the process of solving for *x*. **(CC.7.EE.4a)**

- **Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. (MA10-GR.7-S.2-GLE.2-EO.c.ii)**
  - Refer to the two previous standards, **(MA10-GR.7-S.2-GLE.2-EO.c.i) and (MA10-GR.7-S.2-GLE.2-EO.c.i)** for examples.
- **Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where *p, q,* and *r* are specific rational numbers. (MA10-GR.7-S.2-GLE.2-EO.c.iii)**
  - In this standard students are given problems like: "Mary is buying two pens and a number of notebooks for her school semester. Each pen cost $1.50 and each notebook costs $1.80. If she only has $10 to spend, how many notebooks can she buy?" **(CC.7.EE.4b)**
  - Students identify "only has $10 to spend" as the constraint and recognize that only a few possible solutions satisfy the constraint. They represent the number of notebooks with *x* and solve the inequality $1.8x + 2(1.5) \le 10$ either systematically or by "undoing" the operations. **(CC.7.EE.4b)**
  - From the context of finding the number of notebooks bought, students know that the solution set is limited to positive whole numbers. They conclude that the solutions are $x \in \{1, 2, 3\}$ and exclude non-whole number values, thus representing the entire set of solutions. **(CC.7.EE.4b)**
  - Students also represent the solution as points on the number line.



(CC.7.EE.4b)

  - In solving inequalities, students are careful to consider if they have infinitely many or a finite number of solutions. For example, "A man fills his car with 30 gallons of gas at a station. If the car uses up a gallon for every 28 miles traveled, how far can he get before needing to get fuel again?" Students recognize that volume and distance are continuous quantities and predict that the solution to the inequality contains infinitely many values. **(CC.7.EE.4b)**
- **Graph the solution set of an inequality and interpret it in the context of the problem. (MA10-GR.7-S.2-GLE.2-EO.c.iv)**
  - When asked the problem in the context of a map around a gas station, students may interpret the solution set as a set of points within a circle of radius 840 miles with the gas station at the center or assuming the car keeps to the roads, identify all points on marked roads within and including the radius. **(CC.7.EE.4b)**

(CC.7.EE.4b)

- Another example is: A dog, Teddy, is tied to a tree by a leash with an inelastic part 1 foot long and an elastic part 5 feet long, which is extendable to twice its length. Mary is playing catch with the dog. **(CC.7.EE.4b)**
  1) How far away from the tree can a ball be thrown so that the dog is able to reach it?
  2) Ten dog biscuits were scattered around the tree at these distances away: 1 foot, 3 yards, 6 feet, 10 feet, 5 yards, 8 feet, 14 feet, 16 feet, 24 feet, 36 feet. Which dog biscuits are out of Teddy's reach?
- A common error in these problems is to either **(CC.7.EE.4b)**
  b) Change the sign when *adding* a negative number to both sides of the inequality, or
  c) Forget to change the sign when multiplying or dividing by a negative number after a sequence of steps. To avoid making these mistakes, students learn to substitute a value from the solution set and check if it gives a true statement after they solve the inequality.

Critical Language: includes the Academic and Technical vocabulary, semantics, and discourse which are particular to and necessary for accessing a given discipline. EXAMPLE: A student in Language Arts can demonstrate the ability to apply and comprehend critical language through the following statement: *"Mark Twain exposes the hypocrisy of slavery through the use of satire."*	
A student in ______________ can demonstrate the ability to apply and comprehend critical language through the following statement(s):	*When generating an equivalent inequality I need to reverse the inequality if I multiply or divide by a negative, for example 25 < 32 but if I multiply both sides by -1, the new inequality becomes -25 > -35, notice the inequality sign had to change to keep it true.*
Academic Vocabulary:	Identify, interpret, apply, solve, fluently, compare, graph, addition, subtraction, multiplication, division, equal, correctness, reverse
Technical Vocabulary:	Equivalent, expressions, inequalities, properties of operations, factoring, expansion, arithmetic solution strategy, algebraic solution strategy, arithmetic operations, algebraic equations, algebraic manipulations, operation, both sides, negative number,