Unit Title	Being Rational in an Irrational World	**Length of Unit**	5 weeks
Focusing Lens(es)	Representation Interpretation	**Standards and Grade Level Expectations Addressed in this Unit**	MA10-GR.7-S.1-GLE.2 MA10-GR.7-S.2-GLE.2
Inquiry Questions (Engaging-Debatable):	• What does it mean to "be in the red?" (MA10-GR.7-S.1-GLE.2-RA.1) • Do two negatives always make a positive?		
Unit Strands	The Number System, Expressions and Equations		
Concepts	Sum, distance, positive, negative, direction, additive inverse, subtraction, rational numbers, absolute value, difference, number line, decimal, equivalence, terminating decimal, repeating decimal, fraction, numerator, denominator, multiplication, division, distributive property, signed numbers, products, quotients, integers,		

Generalizations **My students will Understand that…**	**Guiding Questions** **Factual**	**Conceptual**
Every quotient of integers produces rational numbers provided that the divisor is not zero. (MA10-GR.7-S.1-GLE.2-EO.b.iii)	What is an integer? What is a rational number? How can you show 1/7 is rational? What are examples of irrational numbers?	Why is p/0 undefined? Why does the quotient of integers produce a rational numbers provided that the divisor is not zero?
Mathematicians express rational numbers in fractional form as a decimal equivalent that either terminates or eventually repeats (MA10-GR.7-S.1-GLE.2-EO.b.iv)	How do calculate the decimal equivalent for a fraction?	Why do some fractions convert to terminating decimals and others repeating decimals? (MA10-GR.7-S.1-GLE.2-IQ.4)
Mathematicians interpret the sum of rational numbers, $p + q$, as a number located a distance $\|q\|$ from p, in the positive or negative direction (MA10-GR.7-S.1-GLE.2-EO.a.iii)	How can number lines help visualize the possible answers to an addition problem?	When adding, why is it possible for the sum to be smaller than the initial quantity? How do you determine whether to move in the positive or negative direction from an initial quantity when adding?
Mathematicians use additive inverses to interpret subtraction of rational numbers as adding the additive inverse, $p – q = p + (-q)$. (MA10-GR.7-S.1-GLE.2-EO.a.iv)	What are additive inverses? What are situations in which opposite quantities combine to make 0? How can you use additive inverses to subtract rational numbers?	Why do additive inverses have a sum of zero? Why is subtraction of rational numbers equivalent to adding the additive inverse, p – q = p + (-q)?

Mathematicians represent the absolute value of the difference between to rational numbers as the distance between the numbers on a number line. (MA10-GR.7-S.1-GLE.2-EO.a.vii)	How can you represent the distance between two numbers on a number line? If you know the absolute value distance between two numbers, how can you determine if the difference between the two numbers is positive or negative?	Why is $\|p – q\|$ equivalent to $\|q – p\|$?
Fraction multiplication and division extends to rational numbers. (MA10-GR.7-S.1-GLE.2-EO.b)	What are real world examples of multiplication of rational numbers, including negative rational numbers? How does the distributive property help us understand (-1)(-1) = 1 and the rules for multiplying signed numbers and quotients such as –(p/q) = (-p)/q = p/(-q)?	Why is a negative number multiplied by a negative equal to a positive? How do the properties of operations for fraction multiplication extend to rational numbers? (MA10-GR.7-S.1-GLE.2-IQ.1)

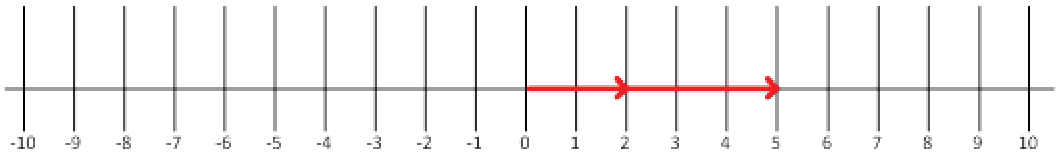
Key Knowledge and Skills: **My students will…**	*What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.*

- **Describe situations in which opposite quantities combine to make 0. (MA10-GR.7-S.1-GLE.2-EO.a.i)**
  - Students learn that that the *Additive Inverse property* states that for any number *n*, *n*+ ¯*n* = 0. The numbers *n* and ¯*n* are called *additive inverses*. Thus, opposite numbers are additive inverses. For example, 34.53 and ¯34.53 are additive inverses because they sum to 0. **(CC.7.NS.1a)**
  - Students are asked to come up with contextual examples of additive inverses from their everyday experiences. They may mention situations where actions are “undone” such as: **(CC.7.NS.1a)**
    1. A board game in which a player advances 5 spaces but then is forced to move back 5 spaces, resulting in no change in position (no advancement) on the board.
    2. The temperature rises by 20 degrees but then decreases by 20 degrees later at night.
    3. Earning $50 for babysitting and then spending $50 on clothes.

(CC.7.NS.1a)

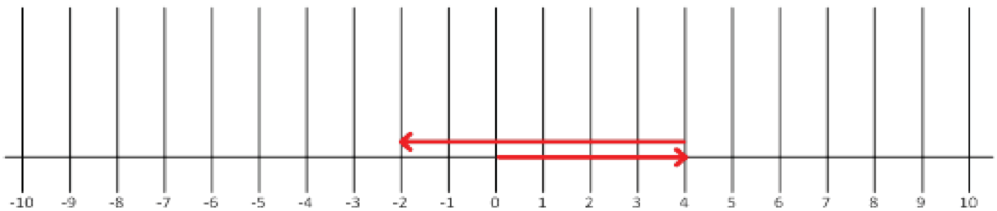
- **Understand *p* + *q* as the number located a distance |*q*| from *p*, in the positive or negative direction depending on whether *q* is positive or negative (MA10-GR.7-S.1-GLE.2-EO.a.iii)**
  - Tasks do not have a context. **PARCC**
  - Tasks are not limited to integers. **PARCC**
  - Tasks involve a number line. **PARCC**
  - Tasks do not require students to show in general that a number and its opposite have a sum of 0. **PARCC**
    - Students use the number line to interpret the sum of two rational numbers *p* + *q*, for which *p* is positive and *q* can be either positive or negative. Addition of rational numbers can be modeled by drawing two vectors “head to tail” [17]. A vector is a ray, (for a definition of a ray see Standard 4.G.1 in the Shapes and Angles LT). Vectors convey distance (indicated by their length) as well as direction (indicated by the direction in which the arrow points). A positive number *a* can be modeled by a ray with length *a* pointing toward the right. A negative number *b* can be modeled by a ray with length *b* pointing toward the left. **(CC.7.NS.1b)**

- The examples below show different vector diagrams on a number line for the cases in which *q* is positive and negative. Note that rational numbers represented as vectors begin at 0 on the number line. **(CC.7.NS.1b)**
  - Case 1: both *p* and *q* are positive, e.g., *p* = 2, *q* = 3; *p* + *q* = 2 + 3**(CC.7.NS.1b)**
- Starting from 0, a student draws a vector whose tail begins at 0 and has length 2. The first vector's head (the arrowhead end) is at 2. Then, from 2, students draw a vector whose tail begins at 2 and has length 3. This vector's head ends at 5, so the two vectors, added, represent 2 + 3 = 5.



(CC.7.NS.1b)

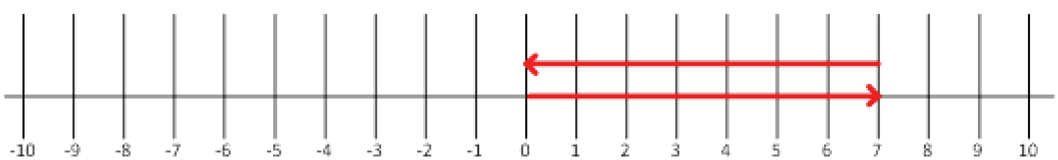
  - Case 2: *p* is positive and *q* is negative, e.g., *p* = 4, *q* = -6; 4 + -6
- As before, a student draws a vector whose tail is located at 0, and whose head is at 4. The second vector, representing the value *q*, points to the left because the number is negative. The student therefore draws a vector whose tail is located at 4 (the head of the first vector), points to the left, has length 6, and therefore ends at -2. **(CC.7.NS.1b)**



(CC.7.NS.1b)

  - The vector diagram above illustrates that 4 + -6 = -2.
- From the vector diagrams, students can also see that if *q* is positive, then *p* + *q* can be interpreted as the number located a distance of |*q*| to the right of *p*. If *q* is negative, then*p* + *q* can be interpreted as the number located a distance |*q*| to the left of *p*.

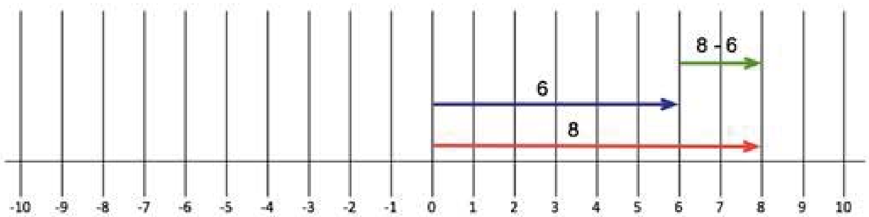
- **Show that a number and its opposite have a sum of 0 (are additive inverses) (MA10-GR.7-S.1-GLE.2-EO.a.iv)**
  - Tasks require students to produce or recognize real-world contexts that correspond to given sums of rational numbers. **PARCC**
  - Tasks are not limited to integers. **PARCC**
  - Tasks do not require students to show in general that a number and its opposite have a sum of 0. **PARCC**
    - Using a vector diagram, students show that if *p* and *q* are additive inverses, then the vectors are the same length but point in opposite directions, and that vector *q* always ends up at 0. The vector diagram below shows that 7 + -7 = 0. **(CC.7.NS.1b)**



(CC.7.NS.1b)

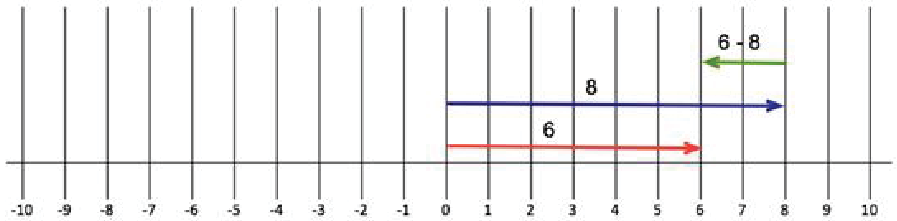
- Students use their vector diagrams to describe the sum of rational numbers in a real world context. For example, *p* + *q* can be interpreted as how far a person walks down the street if they walk *p* units and then walk another *q* units. Students interpret 4 + -6 as a person walking 4 units to the right and then turning around and walking 6 units back to end up 2 units to the left of where she originally started, represented by -2. **(CC.7.NS.1b)**
- Students deepen their flexibility in adding rational numbers, through challenges to draw and interpret vector diagrams for *p* + *q* in which: **(CC.7.NS.1b)**
  1) *p* is negative and *q* is positive; **(CC.7.NS.1b)**
  2) *p* is negative and *q* is negative. **(CC.7.NS.1b)**

- **Understand subtraction of rational numbers as adding the additive inverse, *p* – *q* = *p* + (–*q*) (MA10-GR.7-S.1-GLE.2-EO.a.vi)**
  - Students build their understanding of subtraction of rational numbers by interpreting the difference of two rational numbers, *p* – *q*, using the number line. **(CC.7.NS.1c)**
  - Students can use vector diagrams to model subtraction and show that for two rational numbers *p* and *q*, *p* – *q* = *p* + (-*q*) [17]. In Standard 7.NS.1.b earlier in this LT, addition was modeled as placing a second vector's tail at the first vector's head and finding where the second vector's head lands. **(CC.7.NS.1c)**
  - Subtraction can be regarded as finding the difference between two numbers. Using vectors, subtraction can be modeled as *comparing* two vectors *p* and *q*. At first, this can be done by placing the tails of the two vectors together and asking the question: "How would one extend a vector from the head of *q* to the head of *p*?" The length and direction of that vector would be the result of the subtraction. **(CC.7.NS.1c)**
  - However, it is critical to be consistent with the order in which vectors are compared, and that the order always corresponds to the way the subtraction problem is posed. **(CC.7.NS.1c)**
    - Note in the diagrams that follow that the subtrahend is represented by a blue vector, the minuend by red and the difference by green.]
    - For example, to model 8 – 6, students first draw the vectors 8 and 6 (both starting from 0). They then draw the vector that begins where the vector 6 ends and ends where the vector 8 ends, as in the figure below.



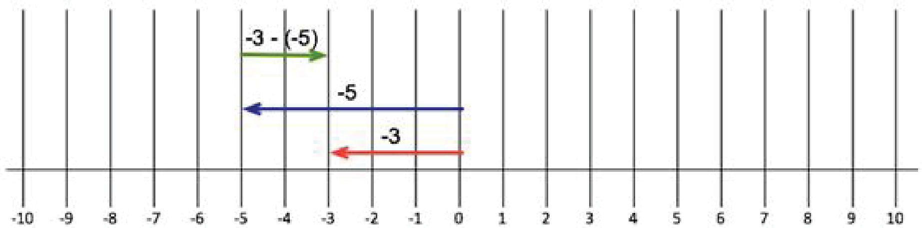
(CC.7.NS.1c)

  - Students reason that the resulting vector is 2 and conclude that 8 – 6 = 2. 6 – 8 = -2 can be found similarly as shown in the figure below. However, students should recall that subtraction, unlike addition, is *not* commutative. Therefore it is critical that students recognize that constructing the vector that represents the difference in a subtraction problem must take into account the order of the numbers in the subtraction statement. Vector models can help reinforce student understanding of subtraction in comparison to addition. **(CC.7.NS.1c)**
  - In the figure below, the vectors representing 6 and 8 are both drawn as starting at 0. However, the vector that represents the difference must again be drawn beginning with the head of the subtrahend vector, and its head must be located at the head of the minuend vector. In this problem (6 – 8), this means that the difference vector begins at 8 and ends at 6. Thus, its length is 2, but its direction is negative, verifying that 6 – 8 = -2. **(CC.7.NS.1c)**



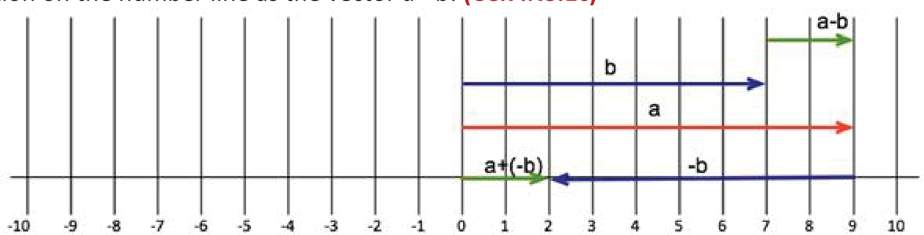
(CC.7.NS.1c)

- Students use this vector comparison model to solve difficult problems such as -3 – (-5). Using the vector subtraction model, the difference vector is shown to be a positive vector with magnitude 2, as seen in the figure below. **(CC.7.NS.1c)**



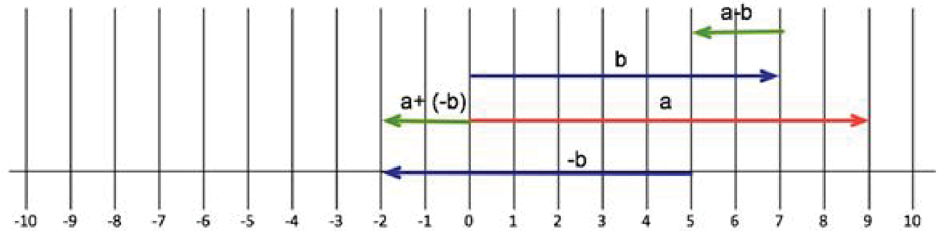
(CC.7.NS.1c)

- The vector model for subtraction can assist students in reasoning about numerical problems that involve subtracting negative numbers. Using these models, they use rules for vectors that, if used consistently, provide a framework to account for both directionality and magnitude of directed numbers (positive and negative numbers). Another important way students make sense of the addition and subtraction of negative and positive numbers is through the application of the additive inverse property (see Standard 7.NS.1.a in the Integers, Number Lines and Coordinate Planes LT). **(CC.7.NS.1c)**
- Students are challenged to explain why -3 – (-5) = -3 + 5 = 2. They use vector models to show that *a* – *b* = *a* + (-*b*) by showing that comparing the difference of the lengths of vectors *a* and *b* is equivalent to adding the opposite vector of *b* to the vector *a*. The figure below demonstrates this for the case in which *a* and *b* are positive and *a* > *b*. For the figure below, students note that after drawing vector -*b* from the head of vector *a*, the head of vector -*b* always ends at the same location on the number line as the vector *a* - *b*. **(CC.7.NS.1c)**



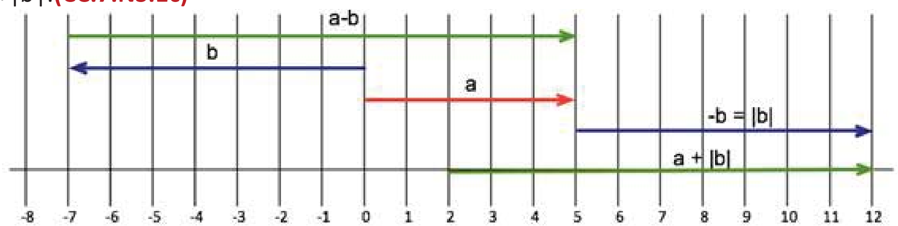
(CC.7.NS.1c)

- Students work through cases of different values for *a* and *b*, and demonstrate to themselves that *a* – *b* is equivalent to *a* + (-*b*) for different relative values of *a* and *b*. The figure below shows the equivalency of the two models for the case in which *a, b* > 0 and *b* > *a*. **(CC.7.NS.1c)**



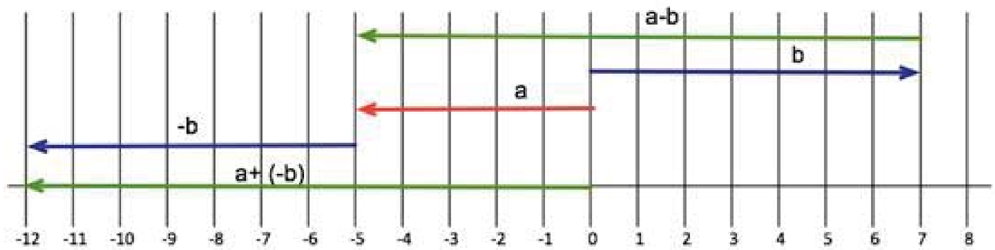
(CC.7.NS.1c)

- The figure below shows the equivalency of the two models for the case in which $a > 0$ and $b < 0$. It provides a visual demonstration of the equivalence of $a - b$ and $a + |b|$.**(CC.7.NS.1c)**



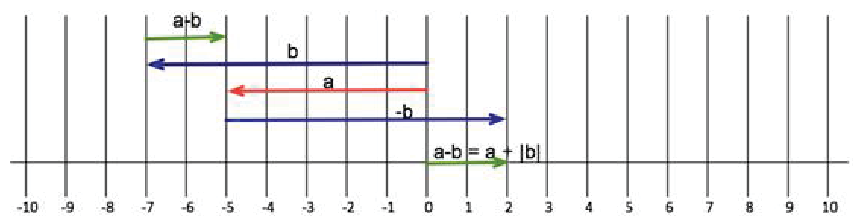
(CC.7.NS.1c)

- The figure below shows the equivalency of the two models for the case in which $a < 0$ and $b > 0$. It provides a visual demonstration of the equivalence of $a - b$ and $a + (-b)$. **(CC.7.NS.1c)**



(CC.7.NS.1c)

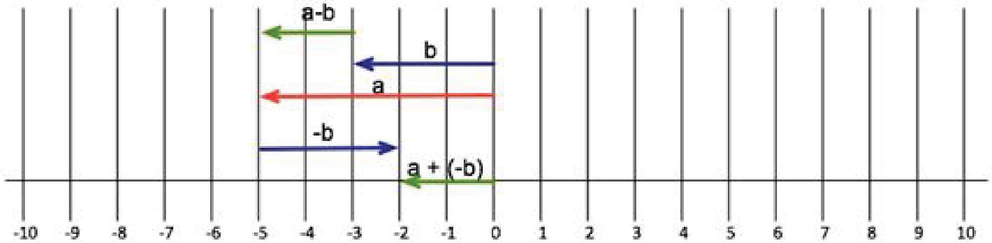
- The figure below shows the equivalency of the two models for the case where $a, b < 0$ and $b < a$. It again provides a visual demonstration of the equivalence of $a - b$ and $a + |b|$ **(CC.7.NS.1c)**



(CC.7.NS.1c)

- **Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. (MA10-GR.7-S.1-GLE.2-EO.a.vii)**

- The figure below shows the equivalency of the two models for the case where *a, b* < 0 and *a* < *b*. It again provides a visual demonstration of the equivalence of *a – b* and *a + (-b)*. **(CC.7.NS.1c)**



(CC.7.NS.1c)

  - Finally, using a number line diagram, students also show that the distance between any two points *p* and *q* is:

    $|p-q| = |q-p|$ **(CC.7.NS.1c)**

- Students are challenged to verify that the distance model and vector model of subtraction work when *p* is negative and *q* is either positive or negative. As before, students avoid the misconception of coding the distance between two points as addition if *p* and *q* are on opposite sides of the number line. **(CC.7.NS.1c)**
- In Standard 6.NS.8 of the Integers, Number Lines, and Coordinate Planes LT, students had experiences finding the distance between coordinate points with the same *x* or the same *y*-coordinate by using subtraction. Students learn that when these coordinate points are located in different quadrants, subtraction (and not addition) is still the correct operation to use. **(CC.7.NS.1c)**

- **Apply properties of operations as strategies to add and subtract rational numbers. (MA10-GR.7-S.1-GLE.2-EO.a.viii)**
  - Tasks are not limited to integers. **PARCC**
  - Tasks may involve sums and differences of 2 or 3 rational numbers. **PARCC**
  - Tasks require students to represent addition and subtraction on a horizontal or vertical number line, or compute a sum or difference, or demonstrate conceptual understanding for example by producing or recognizing an expression equivalent to a given sum or difference. For example, given the sum −8.1+7.4, the student might be asked to recognize or produce the equivalent expression − (8.1−7.4). **PARCC**
    - Students are familiar with properties of addition (commutative, associative, and identity) from their work with the Addition and Subtraction LT. They apply their knowledge of these properties for addition and subtraction (see Standards 1.OA.3, 2.NBT.7, 2.NBT.9, and3.NBT.2 in the Addition and Subtraction LT), order of operations (see Standard 6.EE.2.c in the Early Equations and Expressions LT), and addition and subtraction of integers (see Standards 7.NS.1.b and 7.NS.1.c earlier in this LT) to work with a variety of situations involving adding and subtracting rational numbers, including with parentheses and in different forms including fractions and decimals. **(CC.7.NS.1d)**
      - This becomes particularly useful for students when working with problems that involve several fractions, some of which have common denominators. **(CC.7.NS.1d)**
      - For Example: Terrell's fifth grade class is competing against his sister Tanya's fourth grade class in the finals of the school's annual athletic competition. The score is tied going into the last event, the running long-jump. Each class will have five students perform one jump. Their total jump distances (in yards) will be added to determine the winning class of the event and, hence, the competition. If the distance for each jumper from both classes was recorded as shown in the table below, which class would win the competition? **(CC.7.NS.1d)**

RUNNING LONG JUMP	
Terrell's Class	**Tanya's Class**
$^{12}/_{5}$ yds	$^{8}/_{3}$ yds
$^{5}/_{6}$ yds	$^{5}/_{4}$ yds
$^{4}/_{5}$ yds	$^{2}/_{3}$ yds
$^{7}/_{6}$ yds	$^{7}/_{4}$ yds
$^{9}/_{5}$ yds	$^{4}/_{3}$ yds

(CC.7.NS.1d)

- In order to calculate the total distance jumped by each class, students may use the commutative and associative properties of addition to find each sum, and then compare the total distances. **(CC.7.NS.1d)**

▪ Many situations involving decimals arise, in particular in work with money (see Standard 2.MD.8 in the Time and Money LT). For example, students can determine a store's profit for the month of March given the store's sales revenue and expenditures. **(CC.7.NS.1d)**

March Sales:
Shoes: $417.28
Hats: $223.12
Balloons: $5.17
March Expenditures:
Inventory: $121.12
Rent: $650.00
Utilities: $175.67
Salaries: $212.00
(CC.7.NS.1d)

▪ They add up all of the sales and expenditures to determine that the store took in $645.57 in revenue, but spent $1158.79 on purchases of new inventory and on overhead (rent, utilities, and salaries), and therefore the store's "profit" was $645.57 $1158.79 = -$513.22. **(CC.7.NS.1d)**

- **Understand multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers (MA10-GR.7-S.1-GLE.2-EO.b.i)**
    - o Students have solved problems using the multiplication and division of whole numbers. They extend this and their understanding of negative numbers from Standard 6.NS.6.a in the Integers, Number Lines and the Coordinate Planes LT to apply these operations to problems requiring division or multiplication of non-whole rational numbers. **(CC.7.NS.2a)**
    - o Using their knowledge of positive and negative rational numbers, students learn about products of signed numbers, using mathematical properties. It is important to note that multiplication and division with rational numbers involve understanding of the role of negative numbers, and, in particular, the role of -1 as a multiplier (divisor). Students construct the justifications and demonstrations for multiplication by -1, and then learn that the properties of multiplication and division apply to rational numbers in the same way that they apply to positive numbers and to fractions. **(CC.7.NS.2a)**
    - o Students know that −1 ● 1 and 1 ● (−1) are both equal to −1 by the multiplicative identity property (any number multiplied by 1 is that number). They use the additive and multiplicative identity properties, the multiplicative property of zero, and the distributive property to conclude that (−1) ● (−1) = 1, as shown here:

(−1) ● (−1) = (−1) ● (0 – 1) [using the additive identity property]
= (−1 ● 0) – (−1 ● 1) [using the distributive property]
= 0 – (−1) [using the multiplicative identity property and the
= 1 multiplicative property of zero] (CC.7.NS.2a)

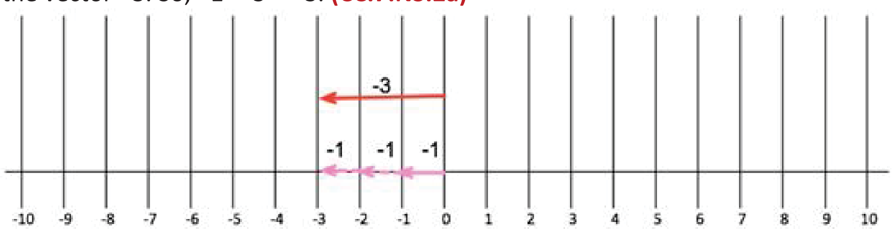
- This can then be generalized to see that the product of any two negative rational numbers is a positive rational number as follows, for any positive rational numbers *a* and *b*:

(–*a*) ● (–*b*) = (–1 ● *a*) ● (–1 ● *b*)
= (–1 ● –1) ● (*a* ● *b*)
= 1 ● (*ab*)
= *ab*
(CC.7.NS.2a)

- Students extend their knowledge of the products of –1's and 1's to the division of those numbers using the axioms of equivalence (see Standard 6.EE.7 in the Linear Equations, Inequalities, and Functions LT) as follows:
  a. Because −1 ● 1 = −1, we know that −1 = $^{-1}/_{1}$ by dividing both sides by 1.
  b. Because −1 ● −1 = 1, we know that −1 = $^{1}/_{-1}$ by dividing of both sides by −1.
  c. Because 1 ● (−1) = −1, we know that 1 = $^{-1}/_{-1}$ by dividing of both sides by −1.
  (CC.7.NS.2a)
- Students use multiple models to represent the product of −1 and a rational number. **(CC.7.NS.2a)**
  1. One model uses the distributive property to show that the product of −1 and a rational number *a* is the number –*a*, as shown below:

     (-1 ● *a*) + (1 ● *a*) = (-1 + 1) ● *a* = 0 ● *a* = 0

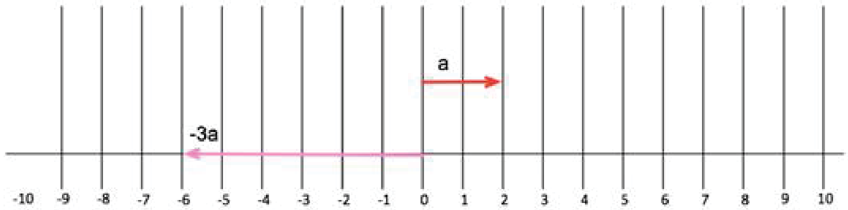
     - So, by subtraction, −1 ● *a* = -(1 ● *a*) = -*a*, by the multiplicative identity property. **(CC.7.NS.2a)**
  2. Students can also use vector diagrams to show that the product of -1 and a rational number *a* is -*a* [1; 4]. For example, to model the product of the vectors -1 and 3, we first draw the vector -1 (a vector with a tail at 0 pointing to the left with a magnitude of 1). We then extend the magnitude of this vector by a factor of 3 to obtain the vector with its tail at 0, pointing to the left, with a magnitude of 3. We see below that this is the vector −3. So, −1 ● 3 = −3. **(CC.7.NS.2a)**



(CC.7.NS.2a)

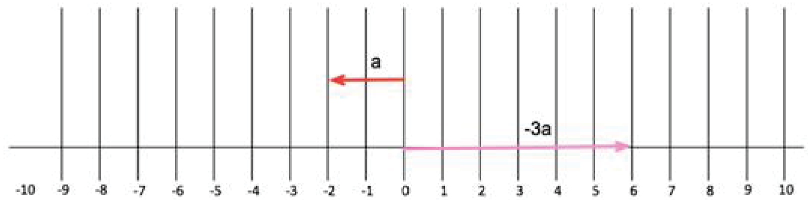
  - Using their knowledge of absolute value (see Standard 6.NS.7.c in the Integers, Number Line and Coordinate Plane LT) and properties of numbers, students extend their understanding of products of −1's and 1's to products of signed numbers. They can also view a negative number *a* as −1 ● |*a*|.**(CC.7.NS.2a)**
    - For example:
    a. −2 ● 3 = (−1 ● |−2|) ● 3 = (−1) ● (2 ● 3) = − 1 ● 6 = −6
    b. 4 ● −5 = 4 ● (−1 ● |−5|) = − 1 ● (4 ● 5) = −1 ● 20 = −20
    c. −5 ● −6 = (−1 ● |−5|) ● (−1 ● |−6|) = (−1 ● −1) ● (5 ● 6) = 1 ● 30 = 30
    (CC.7.NS.2a)

- In a visual vector diagram, if *a* is a vector and *n* is a negative scalar, then the direction of *a*is reversed because of the negative sign of the scalar, and the magnitude changes by a factor of |n|:
  a. If vector *a* > 0, and scalar *n* < 0 **(CC.7.NS.2a)**



(CC.7.NS.2a)

  b. If vector *a* < 0, and scalar *n* <0 **(CC.7.NS.2a)**



(CC.7.NS.2a)

- Students extend their understanding of multiplication and division of fractions (see Section 5 in the Division and Multiplication LT) and apply their knowledge of products of signed numbers to multiply and divide signed rational numbers. They are then able to solve problems involving multiplication and division of rational numbers in a real-world context. **(CC.7.NS.2a)**
- Students recall the three models for multiplication and division as (see Standard 3.OA.3 in the Division and Multiplication LT): **(CC.7.NS.2a)**
  1. Referent transforming
  2. Referent preserving
  3. Referent composing

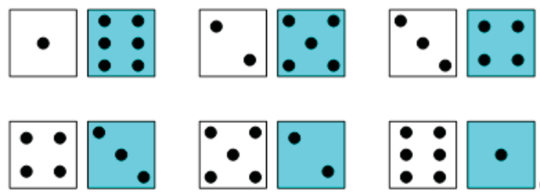
- **Understand integers can be divided, provided the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number; if *p* and *q* are integers, then –(*p/q*) = (–*p*)/*q* = *p*/(–*q*) (MA10-GR.7-S.1-GLE.2-EO.b.iii)**
  - Tasks are not computation tasks but rather require students to demonstrate conceptual understanding, for example by providing students with a numerical expression and requiring students to produce or recognize an equivalent expression. **PARCC**
    - Students are able to associate rational numbers with fractions and the division of integers. They understand that all rational numbers can be written in the form *p/q* where *p* and *q* are integers and $q \neq 0$. In this standard, students explore the converse of this, noting that the division of two such integers always results in a rational number. **(CC.7.NS.2b)**
      - Since students also are familiar with decimals, they can find and write decimal expansions of rational numbers using division. They realize that the two representations are equivalent and express the same number or value. **(CC.7.NS.2b)**
      - For example, students convert and express the following numbers as equivalent:

$$4 = {}^{4}/_{1} = 4 \div 1 = 4.0$$
$${}^{3}/_{2} = 3 \div 2 = 1.5$$

$$^{1}/_{3} = 1 \div 3 = 0.333 \ldots$$

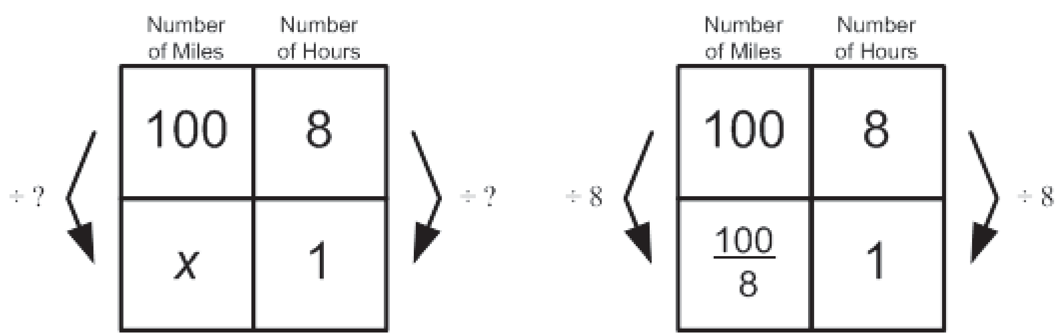
(CC.7.NS.2b)

- From work with rational numbers and their decimal expansions, students notice that every rational number has a decimal equivalent where the decimal either terminates or repeats. They may have worked with decimals in other settings where the decimal was not derived directly from a fraction but they nonetheless informally believe that every decimal also has a fraction equivalent. At this stage, students are introduced to the idea of the existence of decimal expansions that do not terminate or repeat, but they do not need to know how such numbers are derived or any equivalent forms for them. The formal introduction to these numbers, irrational numbers, comes in Standards 8.NS.1 and 8.NS.2 later in this LT. **(CC.7.NS.2b)**
- Students also explore the meaning of rational numbers in real world contexts. For instance, in finding the simple probability of an event, students recognize that the result will always be a rational number between 0 and 1 because the total number of outcomes is always a positive integer and the total number of ways for "success" is a positive integer less than or equal to the total number of outcomes (see Standard 7.SP.5 in the Chance and Probability LT). **(CC.7.NS.2b)**
- For example, students determine the probability of rolling two dice and getting a sum equal to 7. They know that there are 36 ways to roll two dice and write out all the ways that two dice can sum to 7 as shown in the figure below. **(CC.7.NS.2b)**



(CC.7.NS.2b)

- They determine that the probability of rolling a 7 is equal to $^{6}/_{36}$, or $^{1}/_{6}$, which could also be written as 0.166…**(CC.7.NS.2b)**
  - Similarly, students relate ratios to rational numbers through their use of the ratio box [6-8]. For example, in finding the unit rate of a cyclist who rides at a constant pace and covers 100 miles in 8 hours, students set up the following ratio boxes:



  - They know from their prior work with ratio boxes that the co-splitting relationship can be described as division by an integer (8). Since the original quantities were both integers, the result will be a rational number ($^{100}/_{8} = {}^{25}/_{2}$ = 12.5 miles per hour). **(CC.7.NS.2b)**

- **Interpret sums, products and quotients of rational numbers by describing real-world contexts (MA10-GR.7-S.1-GLE.2-EO.a.v, b.ii, b.iii)**
  - Tasks are not computation tasks but rather require students to demonstrate conceptual understanding, for example by providing students with a numerical expression and requiring students to produce or recognize an equivalent expression using properties of operations, particularly the distributive property. For example, given the expression $(-3)(6+\ -4+\ -3)$ the student might be asked to recognize that the given expression is equivalent to $(-3)(6+\ -4)+(-3)(-3)$. **PARCC**
    - Students also have an understanding of ratio and proportion. Ratio and proportion can be viewed as a means to relate their understanding of referent transforming and referent preserving models of multiplication and division. The following problems demonstrate the interrelationships that students learn to use flexibly to solve problems in context. **(CC.7.NS.2a)**
    - **Problem One: Referent Transforming Problems and Ratio Boxes with positive rational numbers (CC.7.NS.2a)**
      - "Aiden needs to buy food for his dog Dusty. The Pet Emporium (TPE) has a 30-lb bag of Dusty's favorite brand on sale for $19.95, while Woofs'R'Us (WRU) has a 40-lb bag of Dusty's favorite brand on sale for $24.95. Which food package should Aiden buy if he wants to get the better deal on Dusty's favorite brand?" **(CC.7.NS.2a)**

Dog food (pounds)	Cost (dollars)
30	19.95
1	0.665

(CC.7.NS.2a)

Dog food (pounds)	Cost (dollars)
40	24.95
1	0.62375

(CC.7.NS.2a)

- If students solved this problem as a pair of division problems, one would have written \$19.95 ¸ 30 = \$0.665 and 24.95 ¸ 40 = \$0.62375. Both solutions would have been obtained using referent transforming division of dollars divided by pounds to produce the unit rate of dollars per pound. **(CC.7.NS.2a)**

▪ **Problem Two: Referent Preserving Problems and Ratio Boxes with positive rational numbers (CC.7.NS.2a)**

- Suppose that a recipe calls for $^2/_3$ cups of shortening and 2 ¼ cups of flour to make a piecrust. For a wedding, the pastry chef needs to scale up the recipe. She would like an easier way to remember the relationship between shortening and flour. The ratio box is a useful tool to organize the work involved. **(CC.7.NS.2a)**

Shortening (Cups)	Flour (Cups)
$^2/_3$	$^9/_4$

(CC.7.NS.2a)

- To simplify the ratio between the two values for shortening and flour to be a ratio of integers, students find a least common multiple (LCM) between the denominators of the fractions (i.e., the least common denominator, or LCD). For this example, the LCD is 12. So, using a form of co-splitting, students multiply both the shortening and the flour by 12 to get 8 cups of shortening and 27 cups of flour. If the original recipe served six, then this new recipe would serve 72 people. **(CC.7.NS.2a)**

Shortening (Cups)	Flour (Cups)
$^2/_3$	$^9/_4$
8	27

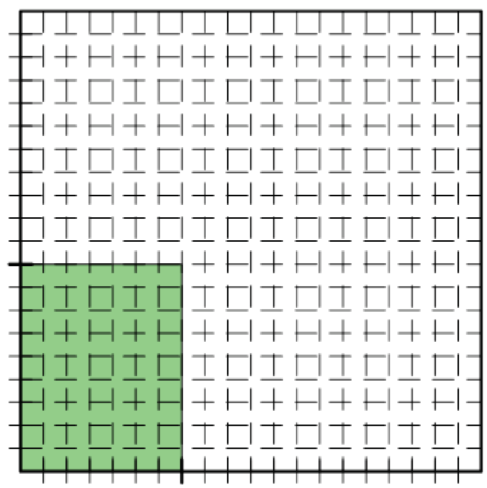
(CC.7.NS.2a)

- Written as a product ($^2/_3$ ● 12 = 8 and $^9/_4$ ● 12= 27), the problem moves vertically in the ratio box, and therefore is referent preserving.
  - Note that this problem builds on division of unit fractions [2] (See Standard 5.NF.7.c in the Division and Multiplication LT), and extends this to division by non-unit fractions, including improper fractions. **(CC.7.NS.2a)**
  - Note to Teachers: Students know fractions as representing division (and vice versa), but they do not explore complex fractions until later in Standard 7.NS.3 in this LT. Therefore, the treatment of “flip the denominator and multiply” cannot be fully explained and understood here. The reason that rote procedure works is explained and can be established within that standard. **(CC.7.NS.2a)**
- Students also extend the third model for multiplication and division to apply to a variety of applications using different types of rational numbers. For instance, students can solve problems using percentages of percentages by modeling them as products using area models. **(CC.7.NS.2a)**

▪ **Problem Three: Referent Preserving Problems with Negative Numbers (CC.7.NS.2a)**

- A meteorologist in Fargo, ND, predicts that this year the lowest temperature will be twice as low the lowest temperature last winter. The lowest temperature last winter was -12.4°F. What is the predicted lowest temperature in Fargo this winter? **(CC.7.NS.2a)**
- Students use their knowledge of products of signed numbers to multiply 2 and -12.4 to obtain 2 ● (-12.4) = -24.8°F. **(CC.7.NS.2a)**

- **Problem Four: Referent Preserving Problems with Negative Numbers (CC.7.NS.2a)**
  - A tank currently has 5,000 liters of liquid in it, and 1,000 liters are removed every day for industrial use. How many liters of liquid were in the tank 3 days ago? **(CC.7.NS.2a)**
  - Students know that the amount of liquid in the tank decreases by 1,000 liters per day (or changes by -1,000 every day). Since the problem asks for the amount of liquid in the tank 3 days ago, they need to multiply that change by –3 to reflect the 3 "sets" of –1,000 liters. So the change in amount of liquid is found by finding the product of –3 and -1,000 to obtain a total of 3,000. **(CC.7.NS.2a)**
  - The full calculation is 5,000 + (-3 ● -1,000) = 5,000 + 3,000 = 8,000. So 3 days ago there were 8,000 liters of water in the tank. **(CC.7.NS.2a)**
- **Problem Five: Referent Composing (CC.7.NS.2a)**
  - Thirty-five percent of a population of fish is speckled, and forty-five percent of those have stripes on their dorsal fins. If the total population of fish in an aquarium is 800 fish, how many of them will be speckled and have stripes on their dorsal fins? **(CC.7.NS.2a)**
  - Students model the problem using a rectangle of area 1. They divide each side into twenty sub-units, each mark representing five percent of the length of the side. They find the mark for 0.35 (35% or 7 units) on one side and 0.45 (45% or 9 units) on the other side, and shade in the product of the area ($^{63}/_{400}$ units). If the population of fish is 800, then students multiply using this result to determine the total number of speckled fish with striped dorsal fins is 126 fish ($^{63}/_{400}$ ● 800). Students recognize that they could also solve the problem by multiplying 0.35 ● 0.45 ● 800. **(CC.7.NS.2a)**

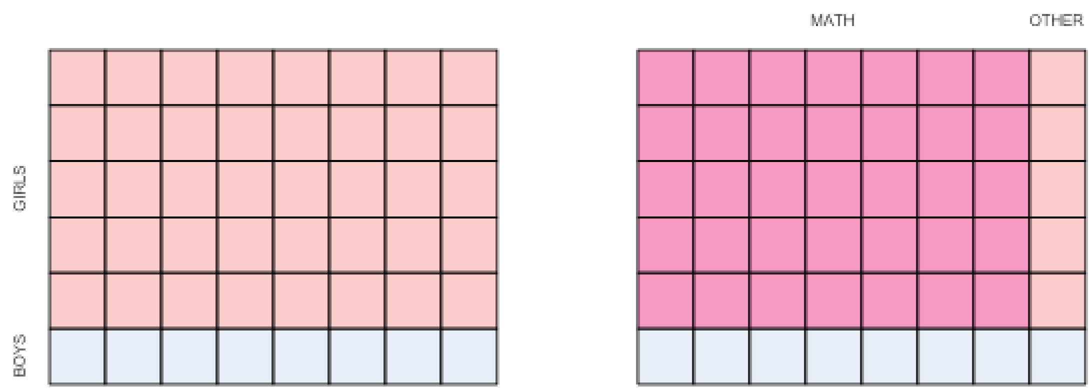


(CC.7.NS.2a)

- **Apply properties of operations as strategies to multiply and divide rational numbers. (MA10-GR.7-S.1-GLE.2-EO.b.iii, b.iv)**
  - o Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$ **PARCC**
    - Students are familiar with properties (commutative, associative, and distributive) of multiplication (see Standard 3.OA.5 of the Division and Multiplication LT). They extend their understanding of these properties to contexts requiring division or multiplication of non-whole rational numbers. **(CC.7.NS.2c)**

- For example, to multiply by mixed numbers, students apply the distributive property of multiplication over addition by additively decomposing the mixed number into its whole number and fractional components (e.g., 6(4 ½) = 6(4 + ½) = 6(4) + 6(½) = 24 + 3 = 27). **(CC.7.NS.2c)**

▪ Students also use the commutative and associative properties of multiplication to simplify multiplication of two rational numbers. Consider the example of finding the number of girls in the fifth grade whose favorite subject is math, given that $^{5}/_{6}$ of the 96 students in that grade are girls, and $^{7}/_{8}$ of the fifth-grade girls say their favorite subject is math. **(CC.7.NS.2c)**



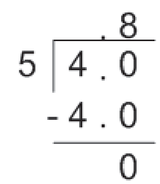
(CC.7.NS.2c)

▪ Students write this problem as the product $(^{5}/_{6})(^{7}/_{8})$. Students first rewrite each fraction (see Standard 4.NF.4.a in the Division and Multiplication LT), resulting in the product $5(^{1}/_{6})*7(^{1}/_{8})$. Then, applying the commutative property, they write the product as $5*7(^{1}/_{6})(^{1}/_{8})$. Next, they apply the associative property to write the product as $5(7)*(^{1}/_{6})(^{1}/_{8})$, which then yields the fraction of girls whose favorite subject is math $(35(^{1}/_{48}) = {}^{35}/_{48})$. Finally, they proceed to find the total number of girls in the fifth grade whose favorite subject is math by multiplying $96*(^{35}/_{48}) = 70$. **(CC.7.NS.2c)**

▪ Students use the commutative and associate properties of multiplication to prove and verify other facts about rational numbers. For instance, they show that:

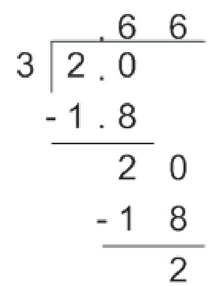
-(*a*/*b*) = -*a*/*b* = *a*/-*b*. First, students rewrite the left-hand side of the equation as
-1(*a1/*b*) = (-1**a*)*1/*b* = -*a**1/*b* = -*a*/*b*. Then, they rewrite the left-hand side again as**
-1(*a1/*b*) = *a*(-1*1/*b*) = *a*(1/-*b*) = *a*/-*b*.**
(CC.7.NS.2c)

- **Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats (MA10-GR.7-S.1-GLE.2-EO.b.v)**
  - o Students learn that rational numbers are numbers that can be expressed as a fraction *a/b*, where *a* and *b* are integers and *b* does not equal 0 (because division by 0 is undefined). From Standard 5.NF.3 in the Equipartitioning LT, they know that *a/b* is equivalent to *a* ÷ *b*, and therefore any rational number's decimal equivalent can be found by dividing *a* by *b*. Students learn by practice that all of these decimal equivalents either terminate in 0's or eventually repeat. **(CC.7.NS.2d)**
    - ▪ Example 1: A rational number that terminates in 0: $^{4}/_{5}$ = .8 **(CC.7.NS.2d)**



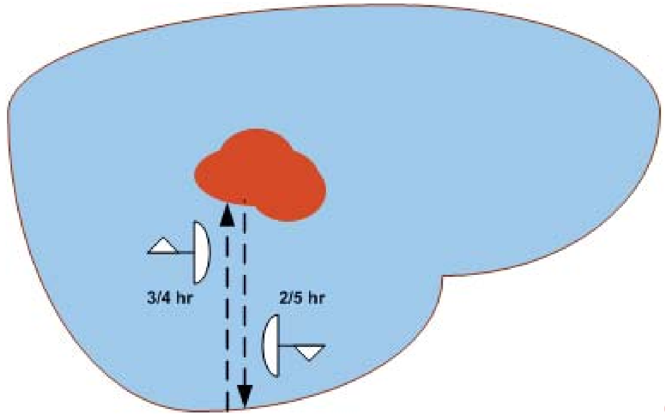
(CC.7.NS.2d)

- Example 2: A rational number that repeats: $^{2}/_{3}$ = .666666… **(CC.7.NS.2d)**



(CC.7.NS.2d)

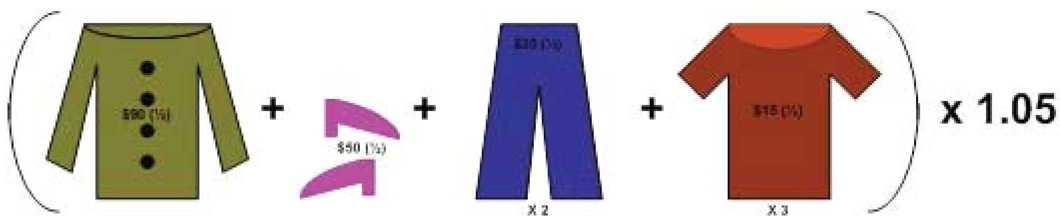
- **Solve real-world and mathematical problems involving the four operations with rational numbers. (MA10-GR.7-S.1-GLE.2-EO.c)**
  - Tasks are one-step word problems. **PARCC**
  - Tasks sample equally between addition/subtraction and multiplication/division. **PARCC**
  - Tasks involve at least one negative number. **PARCC**
  - Tasks are not limited to integers. **PARCC**
    - Students combine their knowledge and skills from prior standards in this LT and other previously referenced standards from the Addition and Subtraction LT as well as the Division and Multiplication LT in order to solve problems involving combinations of the four operations. This is extended to include rational numbers, and in particular division problems where the divisor is a rational number. This creates a *complex fraction*, which is a fraction in which the denominator is itself a fraction. **(CC.7.NS.3)**
      - For Example: The Herms family takes their sailboat to an island in a nearby lake for a picnic. The island is 6 miles from shore. It takes the family $^{3}/_{4}$ hour to sail to the island and only $^{2}/_{5}$ of an hour to sail back to shore with a stronger wind. What was the family's average rate of sailing speed (miles per hour) for the entire trip? **(CC.7.NS.3)**



(CC.7.NS.3)

- Students add the two fractional parts of an hour to determine the total time spent sailing, resulting in $^{3}/_{4} + ^{2}/_{5} = ^{23}/_{20}$ hours. They know the total distance traveled was 12 miles (6 + 6) and apply the formula *distance = rate*time* to find the rate. This gives them an equation that involves a complex fraction, 12 ¸ $^{23}/_{20}$. Students reason that this is not equivalent to finding the compound quotient 12 ¸ 23 ¸ 20 by the order of operations. They may choose to write the equation in one of the following complex fraction forms in order to make that more clear and explicit: $\frac{12}{^{23}/_{20}} = \frac{^{12}/_{1}}{^{23}/_{20}}$. In the form on the right, they rewrote the numerator as a rational number. **(CC.7.NS.3)**
- Students know how to divide rational numbers by integers and recognize multiplication by an integer divided by itself as a special way to represent multiplication by 1, which will not change the value of an expression. They determine that the denominator would become a whole number if multiplied by 20 (or any multiple of 20), so they multiply the complex fraction by $^{20}/_{20}$, which gives $\frac{12/1}{23/20} \cdot {}^{20}/_{20} = \frac{\frac{240}{1}}{23} = {}^{240}/_{23}$. **(CC.7.NS.3)**
  - This procedure can be generalized as follows:
    - Given the complex fraction $\frac{a/b}{c/d}$
      1. Multiply both the numerator and denominator by *d,*
      $$\frac{\frac{a}{b}\cdot d}{\frac{c}{d}\cdot d} = \frac{\frac{a}{b}\cdot a}{c}$$
      2. Multiply both the numerator and denominator of the complex fraction by *b*
      $$\frac{\frac{a}{b}\cdot d\cdot b}{c\cdot b} = \frac{ad}{bc}$$
      3. Students notice this same answer would have resulted from the product of the numerator from the original complex fraction and the reciprocal of the denominator from the original complex fraction (i.e.,$\left(\frac{a}{b}\cdot\frac{d}{c}\right)$. So, for our earlier example, students would find the answer by finding the product $\left(12/1 \cdot 20/23 = {}^{240}/_{23}\right)$ **(CC.7.NS.3)**

- **Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. (MA10-GR.7-S.2-GLE.2-EO.a)**
    - Students solve multi-step word problems that require coordinating among integers, fractions, decimals, and percents. **(CC.7.EE.3)**
        - For example, Susie's favorite store is having a sale where everything is $^1/_2$-off the original marked price. She has \$200 to spend, but she wants to keep 20% of that money for dinner and a movie. At the store, Susie picks out a jacket that is originally marked \$90, shoes that are originally marked \$50, two pairs of pants that are each originally marked \$35, and three shirts that are each originally marked \$15. If there is a 5% sales tax after the sale discount is taken at the register, will Susie be able to purchase everything she picked out and still have saved the amount of money she wanted for dinner and a movie? **(CC.7.EE.3)**
            - Students determine that Susie wants to keep 20% of \$200 or .20 * 200 = \$40. They draw the picture below to visualize how much Susie spends before taxes (everything in the parentheses) and know that adding a 5% sales tax is the same as multiplying the amount spent by 1.05. **(CC.7.EE.3)**



            - Therefore, before taxes, Sally spent (90)($^1/_2$) + (50)($^1/_2$) + (35)($^1/_2$)(2) + (15)($^1/_2$)(3) = 45 + 25 + 35 + 22.50 = \$127.50. **(CC.7.EE.3)**
            - After taxes, Sally has spent \$127.50 * 1.05 = \$133.88. **(CC.7.EE.3)**
            - Therefore, she has \$200 – \$133.88 = \$66.13 left over, which is more than the \$40 she needs for dinner and a movie. **(CC.7.EE.3)**
- **Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. (MA10-GR.7-S.2-GLE.2-EO.b)**
    - Refer to above example from **(MA10-GR.7.S.2-GLE.2-EO.a)**

Critical Language: includes the Academic and Technical vocabulary, semantics, and discourse which are particular to and necessary for accessing a given discipline. EXAMPLE: A student in Language Arts can demonstrate the ability to apply and comprehend critical language through the following statement: *"Mark Twain exposes the hypocrisy of slavery through the use of satire."*	
A student in _______________ can demonstrate the ability to apply and comprehend critical language through the following statement(s):	*I know every rational number written as a fraction has a decimal equivalent that either terminates or repeats.* *I know the distance between 3 and -5 is the same as the distance from -5 and 3, which is the absolute value difference.*
Academic Vocabulary:	Describe, show, apply, convert, solve, strategic, , distance, positive, negative, direction subtraction, fraction, multiplication, division,
Technical Vocabulary:	Sum, additive inverse, rational numbers, absolute value, difference, number line, decimal, equivalence, terminating decimal, repeating decimal, numerator, denominator, distributive property, signed numbers, products, quotients, integers, opposite quantities,